TESTING THE EFFICIENCY OF THE BINOMIAL OPTION PRICING MODEL IN THE INDIAN EQUITY OPTIONS MARKETS FOR THE PERIOD 2010 TO 2015 USING THE NIFTY OPTION

Hemal Thakker
Research Scholar
Pacific Academy of Higher Education and Research University, Udaipur, India
Email: hemal.thakker@kbs.ac.in

Dr A.A Attarwala
Research Guide
Pacific Academy of Higher Education and Research University, Udaipur, India
Email: abbas.attarwala@kbs.ac.in

ABSTRACT

This research article looks to Test the efficiency of the Binomial option pricing Model in the Indian Equity Options Markets for the period 2010 to 2015 using The Nifty Option.

From the time, that Derivative instruments were introduced in the Indian budgetary framework there is much published on the same. This study endeavors to gage the adequacy of choice evaluating model on the most fluid exchanging instrument of the Indian subordinate market: The CNX NIFTY 50. It means to see how the subsidiaries showcases in India functions, essentially concentrating on the Nifty Options Market.

It stays into the hypothetical basics as well as testing its effectiveness. In spite of the fact that the Indian Derivative market is still new, it has the global pattern of proceed with extension, in light of the fact that the different advantages offered by this classification is a lesser known monetary instruments: it helps overseeing remote trade dangers, Flexibility, low exchanging costs, broadening portfolio plausibility and the arrangement methodologies, and the rundown may proceed.

Keywords: Indian Equity Options Markets

INTRODUCTION

It is important to differentiate between an option premium and its theoretical value. As discussed previously, the option premium is the price the option buyer pays to the seller in order to have the right granted by the option, and it is the money the seller receives in exchange for writing the option. The theoretical value of an option, on the other hand, is the estimated value of an option – a price generated by means of a model. It is what an option should currently be worth using all the known inputs, such as the underlying price, strike and days until expiration. These factors often change during an option's lifetime, and some fluctuate in value on a continuing basis throughout any trading session. A pricing model will create theoretical values, but they are just that – theoretical. Specific values for each factor can be used to predict an option contract's theoretical value at a given point in the future.
implied volatility to use, so they must make educated guesses (theoretical values). The implied volatility will then change based upon the supply and demand for the options.

Option traders utilize various option price models to attempt to set a current theoretical value. Models use certain fixed known in the present – factors such as underlying price, strike and days till expiration – along with forecasts (or assumptions) for factors like implied volatility, to compute the theoretical value for a specific option at a certain point in time. Variables will fluctuate over the life of the option, and the option position's theoretical value will adapt to reflect these changes.

Most professional traders and investors who trade significant option positions rely on theoretical value updates to monitor the changing risk and value of option positions and to assist with trading decisions. Many options trading platforms provide up-to-the-minute option price modeling values, and option pricing calculators can be found online at various Web sites

The Binomial Model

The binomial model breaks down the time to expiration into potentially a very large number of time intervals, or steps. A tree of stock prices is initially produced working forward from the present to expiration. At each step it is assumed that the stock price will move up or down by an amount calculated using volatility and time to expiration. This produces a binomial distribution, or recombining tree, of underlying stock prices. The tree represents all the possible paths that the stock price could take during the life of the option.

At the end of the tree -- ie at expiration of the option -- all the terminal option prices for each of the final possible stock prices are known as they simply equal their intrinsic values.

Next the option prices at each step of the tree are calculated working back from expiration to the present. The option prices at each step are used to derive the option prices at the next step of the tree using risk neutral valuation based on the probabilities of the stock prices moving up or down, the risk free rate and the time interval of each step. Any adjustments to stock prices (at an ex-dividend date) or option prices (as a result of early exercise of American options) are worked into the calculations at the required point in time. At the top of the tree you are left with one option price.

Consider a stock whose price is initially S0: We are interested in deriving the current price (f0) of a European call option on the stock. Suppose that the option lasts for time T and that during the life of the option the stock price can either move up from S0 to a new level, S0u; or down from S0 to a new level, S0d (so u > 1; d < 1). Thus the movement of the stock price can be described by the so called one-step binomial tree. At time T, let the payo§ from the option be fu if the stock price moves up, and fd if the stock price moves down. In what follows we price the option by assuming that no arbitrage opportunities exist. First, we set up a portfolio of the stock and the option in such a way that there is no uncertainty about the value of the portfolio at expiration of the option, i.e. at time T: In particular, we consider a portfolio consisting of a long position in shares and a short position in one option. We calculate the value of that makes the portfolio riskless: S0u fu | {z } value of portfolio when S moves up = S0d fd | {z } value of portfolio when S moves down ) = fu fd S0 (u d) : (1) The above equation shows that is the ratio of the change in the option price to the change in the stock price as we move between nodes. The absence of arbitrage opportunities implies that the fair price of any investment is given by the present value of its future payo§. So we should have that the cost of setting up the above portfolio is equal to the present value of its future value. Since the portfolio has no risk, we should use the risk-free interest rate (r) to discount any future payments. In other words, a riskless portfolio must earn the risk-free interest rate. Therefore, we have S0 f0 | {z } cost of the portfolio = (S0u fu) e rT | {z } present value of future payo§ (2) 1 or f0 = S0 (S0u fu) e rT : (2i) Substitute (1) into the above to get f0 = e rT [pfu + (1 p) fd] | {z } expected future value ; (3) where p = e rT u d : (4) It is natural to interpret the variable p given by eq. (4) as the probability of an up movement in the stock price, and the variable 1p as the probability of a down movement in the stock price. Thus, eq. (3) states that the value of the option today is its expected future value discounted at the risk-free interest rate. The
expected stock price at time T is $E(ST) = pS0u + (1 - p)S0d = pS0(u - d) + S0d$ using (4) = $E(ST) = e^{rT}d - uS0(u - d) + S0d$. The above shows that the stock price grows, on average, at the risk-free rate. Therefore, setting the probability of an up movement equal to $p$, is equivalent to assuming that the per annum rate of return on the stock equals the risk-free rate. In a risk-neutral world all individuals are indifferent to risk. They require no compensation for risk, and the expected return on all securities is the risk-free rate. Equation (5) demonstrates that we are assuming a risk-neutral world when we set the probability of an up movement to $p$. This result is an example of an important general principle in option pricing known as risk neutral valuation. The principle shows that it is valid to assume the world is risk neutral when pricing options. However, the resulting option prices are correct not just in a risk neutral world, but in the real world as well.

LITERATURE REVIEW

MacBeth and Merville (1980) compare the Black-Scholes model against the constant elasticity of variance (CEV) model, which assumes volatility changes when the stock prices changes. Empirical evidence of the relationship between the level of stock prices and the rate of volatility is contradictory. Blattberg and Goneses (1974) suggest volatility of the underlying stock is stochastic and random. Rosenberg (1973) suggests that it follows an autoregressive scheme. Black (1976) suggests that the volatility of the underlying stock varies inversely with stock prices. MacBeth and Merville (1980) found that the volatility of the underlying stock decreases as the stock price rises. Their empirical results are also consistent with the results of Geske (1979). Beckers (1980) tested the Black-Scholes assumption that the historical instantaneous volatility of the underlying stock is a function of the stock price, using S&P 500 index options 1972-1977. Beckers (1980) finds the underlying stock is an inverse function of the stock price. Geske and Roll (1984) show that at an original time both in-the-money and out-of-the-money options contain volatility bias. Geske and Roll (1984) conclude, time and money bias may be related to improper boundary conditions, where as the volatility bias problem may be the result of statistical errors in estimation. Yang (2006) finds implied volatilities used to value exchange traded call options on the ASX 200 Index are unbiased and superior to historical instantaneous volatility in forecasting future realised volatility. Literature proposes the Black-Scholes model may underprice options because the tail properties of the underlying lognormal distribution are too small. Rubinstein (1994) illustrates that the implied volatility for S&P 500 index options exerts excess kurtosis. Shimko (1993) demonstrates that implied distributions of S&P 500 index are negatively skewed and leptokurtic. Jackwerth and Rubinstein (1996) show the distribution of the S&P 500 before 1987 exert lognormal distributions, but since have deteriorated to resemble leptokurtosis and negative skewness. Several studies seek to increase the tail properties of the lognormal distribution by incorporating a jump-diffusion process or stochastic volatility. Trautmann and Beinert (1994) estimate parameters of a jump-diffusion process on German capital markets, against the Black-Scholes model. They find option prices generated through a jump-diffusion model are not comparable from those obtained from the Black-Scholes model. Amin and Ng (1993) examine the ability of stochastic volatility models which are derived using ARCH. Amin and Ng (1993) find ARCH models mitigate moneyness and time to maturity bias but not completely. Das and Sundaram (1999) indicate jump-diffusion and stochastic volatility mitigate but do not eliminate volatility bias. Das and Sundaram (1999) identify jump-diffusion and stochastic volatility processes do not generate skewness and extra kurtosis resembled in reality. Buraschi and Jackwerth (2001) develop statistical tests based on instantaneous model and stochastic models using S&P 500 index options data from 1986-1995. Buraschi and Jackwerth (2001) conclude the data is more consistent with models that contain additional risk factors such as stochastic volatility and jump-diffusion.

Fischer Black and Myron Scholes (1973) the actual options prices deviate in certain systematic ways from the values predicted by the formula. Option buyers pay prices that are consistently higher than those predicted by the formula. Option writers, however, receive prices that are at about the level of predicted by the formula. There are large transaction costs in the option market, all of which are effectively paid by option buyers. The difference between the price paid by option buyers and the
value given by the formula is greater for options on low-risk stocks than the options on high risk stocks. Gurdip B. Charles C and Zhiwu (1997) regardless of performance yardstick, taking stochastic volatility into account is the first order importance in improving upon the Black-Scholes formula. To rationalize the negative skewness and excess kurtosis implicit on option prices, each model with stochastic volatility requires highly implausible levels of volatility return correlation and volatility variation. S. McKenzie, D. Gerace and Z. Subedar (2007) the Black Scholes model is relatively accurate. Comparing the qualitative regression models provide evidence that the Black Scholes model is significant at the 1 per cent level in estimating the probability of an option being exercised. All variables of each regression model exert expected signs of economical significance. The results based on a method of maximum likelihood indicate that the factors of the Black-Scholes collectively are statistically significant. The qualitative regression models also illustrates the significance of the Black-Scholes model under a logistic distribution is superior to a lognormal distribution. Indicating that the use of a jump-diffusion approach increases the tail properties of the lognormal distribution increases the statistical significance of the Black-Scholes model. The second stage least squares approach to test significance of the qualitative regression models provides significance at the 1% level. Shyam Lal Dev Pandey and Mihir Das (2013), GAARCH (1,1) and Black-Scholes model can be used for pricing of index (call and put) and stock (put options) in the Indian stock market. The differences between model and actual prices vary based on time effect. GAARCH and BS Model provides better results for put options and call options with lesser volatility and fewer days to expiry. The results of paired sample T-test show that there is no significant difference between the model and market values. J. Orlin Grabbe (1983) has explored a set of inequality-equality constraints on rational pricing of foreign currency options, and has developed exact pricing equations for European puts and calls when interest rates are stochastic. The assumption that relevant variables follow diffusion processes allows us to set up a riskless hedge that uses no wealth, and which therefore must have a zero return in equilibrium. The construction of this hedge yields a partial differential equation whose solution is the European call option value. The put option equations are obtained immediately from the call equations through a put-to-call conversion equation that holds for FX options. Finally, it was shown that for sufficiently high values (low values) of the spot rate relative Black and Scholes (1973) claimed that in many cases their famous model could be used as an approximation to give an estimate of the warrant value. The Chicago Board Options Exchange (CBOE), the first public options exchange, began trading in April 1973, and by 1975, traders on the CBOE were using the model to price and hedge their option positions. Since then, “thousands of traders and investors use the formula everyday”, noted by the Nobel committee (Marsh & Kobayashi, 2000). It was widely used in those personal computer days that Texas Instruments sold a handheld calculator specially programmed to produce Black-Scholes options prices and hedge ratios. Merton (1998) remarked that the influence of the Black-Scholes option theory on finance practice has not been limited to financial options traded in markets or even to derivatives generally. It is also used to price and evaluate risk in a wide array of applications, both financially and nonfinancially. He further remarked that the publication of the option model in 1973 surely helped the development and growth of the listed options and over-the-counter (OTC) derivatives markets. The seminal paper by Stoll(1969) the relationship has been extended in many directions and widely tested, especially for US stock option markets\(^1\). The introduction and success of index options both in the US and in Europe have called for attention of empirical research to these markets. While most of the literature on index options has focused on US markets (e.g. Ackert and Tian(2001), Evnine and Rudd(1985), Kamara and Miller(1995)), since the mid 90s a few contributions have investigated the validity of the PCP in some relatively new European index option markets. As far as we know, only a few recent papers2, propose efficiency tests on European markets and specifically: Capelle-Blancard and Chaudhury (2001) for the French index (CAC40) option market, Mittnik and Rieken(2000) for the German index (DAX) option German and Cavallo and Mammola (2000) for Italian index (Mib30) option market. The empirical literature on the efficiency of European option markets is not only still limited in number of contributions, but also relates – except for the work of Capelle-Blancard and Chaudhury (2001) – on a pre-Euro period, which normally represents the infancy of the index option market under investigation. 3 to the exercise price, American calls (puts) will be exercised prior to

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2. Such as the papers by Capelle-Blancard and Chaudhury (2001).
3. Assuming the Black-Scholes formula for pricing options.
maturity. Hence (for positive interest rates) American FX options have values strictly greater than European FX options. Ramazan G and Aslihan S (2003), Black-Scholes model is not the proper pricing tool in high volatility situations especially for very deep out-of-the-money options. Feed forward networks provide more accurate pricing estimates for the deeper out-of-the money options and handles pricing during high volatility with considerably lower errors for out-of-the-money call and put options. This could be invaluable information for practitioners as option pricing is a major challenge during high volatility periods. For the deepest out-of-the-money options, the Black-Scholes prices overestimate market prices whereas market prices are underestimated for the deeper and near out-of-the-money options. In particular, the performance of the Black-Scholes model in explaining the observed market prices is quite poor for the deepest out-of-the-money options. Emilia Vasilie and Dan Armeaun (2009) the operators take into consideration the moneyness of an option and the duration up to the due term thereof, when they calculate the volatility on account of which they evaluate the option. This is a direct consequence of the fact the Black-Scholes model cannot be applied in its original form: the prices of the financial assets do not follow log-normal distribution laws. David Chappell (1992) One problem with the Black-Scholes analysis, however, is that the mathematical skills required in the derivation and solution of the model are fairly advanced and probably unfamiliar to many economists. For the riskless rate of return one could use as a proxy the T-Bill rate or LIBOR, suitably adjusted to provide an instantaneous rather than annual rate. For the variance rate, standard deviation various possibilities exist for its estimation.

For a long time, option pricing has been conducted in a rather systematic way. First, one makes an assumption about the data-generating process. That is, one assumes a particular stochastic process, which drives the underlying asset price. The most common choice here is the assumption of geometric Brownian motion for the asset price, which was first advocated by Osborne [1959]. Second, one rewrites that process in risk-neutral terms either using arguments about dynamic replication or considering the influence of the price of risk explicitly. Third, the parameters of the model are estimated and finally, fourth, the option is priced.

This systematic way has been sidestepped in only a few cases, most notably concerning the volatility parameter of the Black-Scholes formula. Volatility is the only parameter in the Black-Scholes formula, which is not directly observable. Therefore, practitioners started soon to find the implied value of volatility by fitting the Black-Scholes model to the observed option prices. The number of exchange-traded options was small for a long time after their launch in the early 1970s, and thus the opportunity for any widespread fitting of parameters other than volatility was limited.

This situation changed dramatically towards the end of the millennium. While in the early tests of option pricing such as Rubinstein [1985], the Black-Scholes model was more or less supported, the index options in the US can now no longer be priced with the simple BlackScholes formula. Ever since the stock market crash of 1987, the implied volatilities of index options tend to slope downward across strike prices whereas the Black-Scholes model would require a flat volatility smile. Unfortunately, this is only the general pattern and there are deviations from the monotonically downward-sloping smile. So can the smile for very low strike prices flatten out at a high plateau, exhibiting a tilde-shaped pattern. Also, for days where we observe very high strike prices the implied volatilities can rise again, exhibiting a u-shaped pattern. Finally, the smile patterns described become more pronounced as the time-to-expiration shortens and the smile becomes flatter as the time-to-expiration lengthens.

Tomkins [1998] finds this same pattern in index options written on the Japanese, German, and British markets. Other markets also exhibit volatility smiles: Toft and Prucyk [1997] find that individual stock options often exhibit more gently downward sloping smiles, and Cappa, Chang, and Reider [1998] find that foreign exchange options often exhibit u-shaped smiles. As researchers were trying to find new option pricing models which were able to explain these option prices, some tried to learn more about the stochastic process of the asset from the observed option prices. This was finally possible.
since we can now find as many as one hundred simultaneously traded options on the S&P500 index which all differ from each other in strike price and/or time-to-expiration. Given such plentiful data, it is a small step to ask what those options tell us about the risk-neutral stochastic process of the underlying security. Rubinstein [1994], Derman and Kani [1994], andDupire[1994] have answered this question in a series of papers on a class of models, which Rubinstein [1994] termed implied binomial trees. These implied trees are extensions of the original Cox, Ross, and Rubinstein [1979] binomial trees.

**RESEARCH METHODOLOGY**

**Purpose of study**

To study if the Binomial Model of option pricing can be used for price discovery in India for the Nifty Options

**OBJECTIVES**

This research thesis attempts to analyze how effective is the pricing models used for European options on the Indian Option Market

The study intends to investigate the efficiency of the Option contract of the National Stock Exchange benchmark Index i.e. S & P CNX Nifty

To understand the pricing of the options based on pricing techniques

To extend the existing domain of knowledge about Option Pricing Behavior

**Statement of Hypothesis:**

**Assumption**

Since the underlining asset is the nifty, it is comprised of many different stocks all of which are dividend paying for the purpose

An assumption is made that the nifty does not pay any dividend

The MOBOR is used as the risk free rate

The Indian VIX is used for the volatility of the NIFTY

**Hypothesis testing**

**Assumptions**

There are only two possible prices for the underlying asset on the next day. From this assumption, this model has got its name as Binomial option pricing model (Bi means two)

The two possible prices are the up-price and down-price

The underlying asset does not pay any dividends

The rate of interest (r) is constant throughout the life of the option

Markets are frictionless i.e. there are no taxes and no transaction cost

Investors are risk neutral i.e. investors are indifferent towards risk

**Hypothesis**

H0:- There is no significant difference between the calculated value of the option price by Binomial Model and the Market value of the Nifty Option

H1:- There is significant difference between the calculated value of the option price by Binomial Model and the Market value of the Nifty Option
T-Test

Paired Samples Statistics

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<th>Mean</th>
<th>N</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
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<tr>
<td>Opening Price of Call Value as per the Binomial Model</td>
<td>482.04</td>
<td>1520</td>
<td>580.67</td>
<td>14.89380</td>
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<td></td>
<td>253.10</td>
<td>1520</td>
<td>709.92</td>
<td>18.209</td>
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Paired Samples Test

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<tr>
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<th>Mean</th>
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<th>Std. Error Mean</th>
<th>95% Confidence Interval of the Difference</th>
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Paired Differences

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<th></th>
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<td>1519</td>
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Since the P value to the Level of Significant is Negligible the Null Hypothesis is rejected proving that there is significant difference between the calculated value of the option price by Binomial Option Pricing model and the Market -value of the Nifty Option Pricing

BIBLIOGRAPHY


