

APPLICABILITY AND ACCEPTABILITY OF FUZZY LOGIC IN PROGRAMMING LOGIC

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ABSTRACT

Some data are incomplete, imprecise and ambiguous. There are some methods that work on these data, they solve unstructured problems with incomplete knowledge by developing inferences and answers as humans do. They provide acceptable solutions to problems that are difficult for other methods to solve. That data is referred as fuzzy data and these methods are termed as fuzzy logic. With this paper I want to throw some light on concept, percept, misconception, applicability, acceptability and reliability of fuzzy logic.

Keywords: Incomplete, Unstructured, Inference, Fuzzy, Fuzzy Logic

INTRODUCTION

In real physical world we encounter a number of classes of objects, which do not have precisely defined criteria of membership. For example, class of living being clearly includes man, woman, animals and birds as its members and clearly excludes such objects as rocks, liquids etc. However object like plants have an ambiguous status with respect to class of living beings. The same kind of ambiguity arises in the case of the “class of tall men”, “class of beautiful things”. These are the cases of impreciseness in defining things, but it is the fact of life that imprecise defined things play an important role in human thinking and behavior.

Everything is a matter of degree!

Fuzzy logic allows for shades of gray between the black and white, and in its linguistic form, imprecise concepts like "slightly", "quite" and "very". Specifically, it allows partial membership in a set. It was introduced in 1965 by Prof Lotfi Zadeh at the University of California, Berkley. In a book “Information and Control” on page 338-353, he defined fuzzy set as “a class of objects with the continuum of grades of membership. Such a set is characterized by a membership function ranging between 0 and 1. It is related to fuzzy sets and possibility theory.

Fuzzy logic is derived from fuzzy set theory dealing with reasoning that is approximate rather than precisely deduced from classical predicate logic. It can be thought as the

application side of fuzzy set theory dealing with well thought out real world expert values for a complex problem. Fuzzy logic framework provides a natural way of dealing with the problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables.

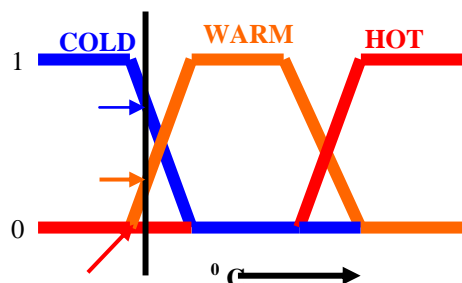


Fig.1. Illustration of particular temperature ranges

In this image, cold, warm, and hot are functions mapping a temperature scale. A point on that scale has three "truth values" — one for each of the three functions. For the particular temperature shown, the three truth values could be interpreted as describing the temperature as, say, "fairly cold", "slightly warm", and "not hot".

LITERATURE REVIEW

The fuzzy logic applications in reality are now innumerable and they range from control system for small domestic appliances or electronic devices (on washing machines or camcorders, for example Matsushita, Hitachi products) to optimized planning of industrial control applications (Kawasaki Steel industry), from efficient and stable control of car engines (Nissan) to controlling of subway systems (Sendai city subway system). These successful applications have convinced almost everyone, not only the Japanese engineers, that fuzzy systems actually "work"[25]. A basic application might characterize sub ranges of a continuous variable. For instance, a temperature measurement for anti-lock brakes might have several separate membership functions defining particular temperature ranges needed to control the brakes properly. Each function maps the same temperature value to a truth value in the 0 to 1 range. These truth values can then be used to determine how the brakes should be controlled [13]. A more sophisticated practical example is the use of fuzzy logic in high-performance error correction to improve information reception over a limited-bandwidth communication link affected by data-corrupting noise using turbo codes. Two decoders analyse the data in parallel, arriving at different likelihood results for the values intended by the sender. Each use as additional data the other's likelihood results, and repeats the process to improve the results until consensus is reached as to the most likely values [30]. The concept of fuzzy mathematics was applied to the damage evaluation of existing reinforced concrete (RC) bridges. The partition rule of evaluation grade was used to divide the evaluation results. The damage state of existing RC bridges was carried out using a double layer fuzzy synthesis evaluation. According to the evaluation results, the double-layer synthesis evaluation of fuzzy for the damage of RC bridge is definitely useful. The evaluation results are also offered as the basis for strategy on the repair and reinforcement of existing RC bridges [26]. Fuzzy logic appears a promising approach to address many important aspects of networks, particularly the traffic control in ATM (Asynchronous

Transfer Mode) network. ATM traffic model and traffic control using fuzzy Controllers are first simulated using MatLab [40]. The motivations for fuzzy logic control (FLC) are illuminated by exploring the benefits obtained by application designers through its use. A context for this exploration is set with a discussion of the characteristics of control policies and of the general attributes of Fuzzy Logic Controller [04]. Using pattern recognition, it is analysed that there is a relation between biochemistry indexes and brain jam, and it is also pointing towards causes of the brain jam. The study has important significance for preventing and diagnosing the brain jam [39]. Appraising classroom teaching qualities is one part of theory of educational measure. To appraise classroom teaching qualities, we often use words like “good”, “fair”, “mediocre”, or “bad”, but these are blurred words. Fuzzy mathematics is an effective way to make a quantitative appraisal of teaching qualities. Also, fuzzy synthetic judgement is combined with fuzzy recognition to give a way to appraise the classroom teaching qualities of a teacher [38]. To obtain the fair price for collective insurances consisting in whole life annuities contracts whose amounts can be either fixed or indexed and to determine the insurance solvency cost, which reflects the amount to be surcharged on pure premiums, for both analysis the main mathematical instruments to be used is the concept of fuzzy random variable (f.r.v.) in which its realizations were fuzzy subsets[16]. This paper presents a hybrid system controller, incorporating fuzzy controller with vector-control method for induction motors. The vector-control method has been optimized by using fuzzy controller. High quality of the regulation process is achieved through utilization of the fuzzy logic controller [36]. It is an effective approach to combine state signal processing with fuzzy mathematics for recognizing fuzziness and randomness clearly. This paper shows how the fuzzy membership functions are obtained from the result of signal processing of fault diagnosis in electrical equipment. The effectiveness of the method was proved by the use of fuzzy logic[22]. To model the uncertainties of data inputs and to provide an approach to risk analysis, Fuzzy Mathematics of Finance, is being used. The goal is to capture and quantify the qualitative information of the project coordinators using linguistics variables and model the uncertain cash flow and discount rates as triangular fuzzy numbers [01]. Fuzzy Logic provides a different way to approach a control or classification problem. This method focuses on what the system should do rather than trying to model how it works. One can concentrate on solving the problem rather than trying to model the system mathematically. In General, the employment of fuzzy logic might be helpful, for very complex processes, when there is no simple mathematical model for highly nonlinear processes [23]. A new body of research at the intersection of economics and psychology is about making financial decisions. The new research is finding that individuals often have self-control problems, overly optimistic expectations, and heuristics that do not square well with traditional concepts of rationality. These “behavioral” aspects of financial decision-making have been shown a new area of research that is fuzzy logic [35]. One of the most important aspects in the invention of fuzzy mathematics, was the invention of obtaining an effective way for modeling badly defined phenomena appearing over and over in the everyday life [21]. Presently, the practical applications of fuzzy logic belong more to business administration and engineering than to economics [31]. This paper presents an expert system aimed at evaluating firms and business units. It makes use of fuzzy logic and integrates financial, strategic, managerial aspects, processing both quantitative and qualitative information [24]. Building on insights from basic research for improving financial decision making, biased perceptions of inter temporal tradeoffs are relatively easy

to identify, compared to biased or mistaken expectations. Treating biased perceptions of inter temporal tradeoffs fuzzy mathematics is employed [34]. Investment decision in assets with a high degree of "know-how" especially under uncertainty is an important matter for policy-maker and enterprise managers. In his paper, Saeid H. Kashani developed a new panoramic vision using "fuzzy logic" methodology to show a powerful association between the real and fuzzy estimated data, and explain the behavior of investment decision [33]. Fuzzy logic is possibly the greatest intellectual breakthrough of a lifetime and most certainly has a place in the social sciences--in fact, it is utterly essential because people are FAR more complex than can be described with a logic that only admits yes or no, true or false, answers to questions[07]. This paper makes use of fuzzy logic and integrates financial, strategic, managerial aspects, processing both quantitative and qualitative information at evaluating firms and business units. The system is used for rating, ranking and pricing firms as well as for assessing the impact of managers' decisions on value creation and as a tool of corporate governance. Fuzzy logic seems to be a reliable tool for describing the value of a firm, since the complexity of real-life situations is handled through "vague" variables and "vague" interactions, which better replicate human mind as well as economic phenomena. Also, a fuzzy approach, unlike classical ones, seems to be capable of integrating qualitative and quantitative analysis [15]. Fuzzy Logic (FL) is a powerful tool that has been used in diverse fields of applications ranging from engineering to business or finance. This paper applies the tools of FL to design the integration of an FL model to CG that will rate the level of application of CG [17]. Fuzzy set theory can be used in an uncertain economic decision environment to deal with the vagueness of the human thought. It has the capability of representing vague data and allows mathematical operators and programming to apply to the fuzzy domain. The theory is primarily concerned with quantifying the vagueness in human thoughts & perceptions. Fuzzy logic allows decision making with estimated values under incomplete / uncertain information[03]. In this research fuzzy logic theory is applied to measure the consumption values of Chinese consumers purchasing imported fruit. Results demonstrate that fuzzy logic is an effective approach to quantifying the consumption values that consumers pursue in a give context [37]. Fuzzy set theory and fuzzy logic are a highly suitable and applicable basis for developing knowledge-based systems in medicine for tasks such as the interpretation of sets of medical findings, syndrome differentiation in medicine, diagnosis of diseases ,real-time monitoring of patient data[27]. Using the state-of-the art and especially in the language of technologists and management researchers, the proposed book is an excellent blend of the fuzzy mathematics, information technology and operations management [05]. This book explores the integration of fuzzy sets with other decision support and modeling disciplines, such as multi criteria decision aid, neural networks, genetic algorithms, machine learning, chaos theory[29]. The book aims at surveying results in the application of fuzzy sets and fuzzy logic to economics and engineering. Important topics covered are fuzzy input-output analysis; fuzzy mathematics of finance; fuzzy PERT. No previous knowledge of fuzzy sets is needed. The mathematical background is assumed to be elementary calculus [15].

Definitions

Let X be a class of space of points with a generic element of X denoted by x
so that $X = \{ x \}$.

A fuzzy set A in X is characterized by a membership function $f_A(x)$ which associates with each point in X a real number in the interval [0,1], with the values of $f_A(x)$ at x representing the “grade of membership” of x in A. The nearer the value of $f_A(x)$ to unity, the higher the grade of membership of x in A and the nearer the value of $f_A(x)$ to zero, the lower the grade of membership of x in A. (Zadeh, “Fuzzy Sets”, “II. Definitions”, 1995). Although the membership function of a fuzzy set has some resemblance to a probability function but in fact, fuzzy set is completely non statistical in nature. A fuzzy set is empty iff membership function is identical zero on X.

Two fuzzy sets A and B are equal, written as $A = B$ if and only if

$$f_A(x) = f_B(x) \quad (\text{for all } x \text{ in } X)$$

The complement of a fuzzy set A is denoted by A' and is defined by

$$f_{A'} = 1 - f_A$$

The containment of set A in Set B means A is subset of B iff

$$f_A \leq f_B$$

The union of two fuzzy sets A and B is a smallest fuzzy sets C which is written as

$$C = A \cup B \text{ such as } f_C = f_A \vee f_B.$$

$$\text{and } \text{Max}[f_A, f_B] \geq f_A \text{ and } \text{Max}[f_A, f_B] \geq f_B$$

Suppose D is any fuzzy set which contains both A and B then

$$f_D \geq f_A \text{ and } f_D \geq f_B,$$

hence, $f_D \geq \text{Max}[f_A, f_B] = f_C$, it implies that C is a subset of D

The intersection of two fuzzy sets A and B can be defined as the largest fuzzy sets D which is contained both in A and B. the membership function can be defined as

$$f_D \geq \text{Min}[f_A, f_B] = f_C$$

The membership function of the union is comprised of curve segments 1 and 2; that of the intersection is comprised of segments 3 and 4.

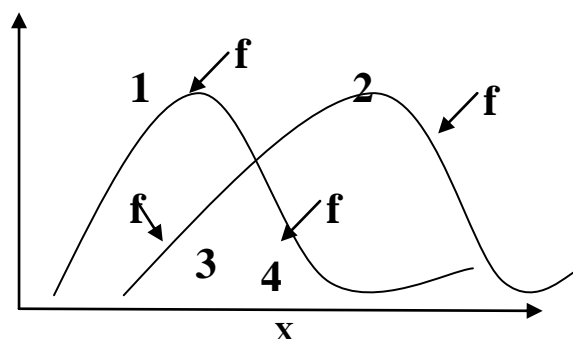


Fig. 2. Union and intersection of fuzzy sets

Some properties of fuzzy sets are defined for three fuzzy sets A, B, and C.

1. Associativity:- fuzzy sets has the associative property, that is,

$$A \cup (B \cup C) = (A \cup B) \cup C$$

2. Distributivity:- fuzzy sets has the distributive property, that is,

$$C \cup (A \cap B) = (C \cup A) \cap (C \cup B)$$

$$C \cap (A \cup B) = (C \cap A) \cup (C \cap B)$$

3. De Morgan's law:- fuzzy sets follow the De Morgan's law, that is,

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

4. Commutativity:- fuzzy sets has the commutative property, that is,

$$A \cap B = B \cap A$$

$$A \cup B = B \cup A$$

Fuzzy relation :- A relation is defined as a set of ordered pairs and in context of fuzzy sets, a fuzzy relation in X is a fuzzy sets in the product space $X * X$. Fuzzy relation has a natural extension to fuzzy sets and plays an important role in the theory of such sets and their applications like ordinary sets

Formal Fuzzy Logic

In application, the programming language Prolog is used to implement fuzzy logic with a database of "rules" which are queried to deduct logic. This sort of programming is known as logic programming. A membership function is a mathematical function which defines the degree of an element's membership in a fuzzy set.

Below is the membership function for tallness

$$\text{tall}(x) = \left\{ \begin{array}{ll} 0, & \text{if height}(x) < 5\text{ft}, \\ (\text{height}(x)-5\text{ft})/2, & \text{if} \\ 5\text{ft} \leq \text{height}(x) \leq 7\text{ft}, \\ 1, & \text{if height}(x) > 7\text{ft} \end{array} \right\}$$

Consider, if a person is less 4'9", then he has a membership value of 0.0 and thus is not a member of the set tall. If a person is 7'6", then he has a membership value of 1.0 and thus is a member of the set tall. And, if a person is 5'5", then he has a membership value of 0.21 and is a partial member of the set tall.

How fuzzy logic is applied

Fuzzy logic usually uses IF/THEN rules, or constructs that are equivalent, such as fuzzy associative matrices. Rules are usually expressed in the form:

IF variable is set THEN action

E.g. A simple temperature regulator that uses a fan might look like this:

IF temperature is very cold THEN stop fan

IF temperature is cold THEN turn down fan

IF temperature is normal THEN maintain level

IF temperature is hot THEN speed up fan

There is no "ELSE". All of the rules are evaluated, because the temperature might be "cold" and "normal" at the same time to differing degrees.

The AND, OR, and NOT operators of Boolean logic exist in fuzzy logic, usually defined as the minimum, maximum, and complement; they are called the Zadeh operators, as they were first defined in Zadeh's original papers. So for the fuzzy variables x and y:

$$\text{NOT (X)} = (1 - \text{truth(X)})$$

$$\text{X AND Y} = \text{minimum (truth(X), truth(Y))}$$

$$\text{X OR Y} = \text{maximum (truth(X), truth(Y))}$$

Other operator called hedges can be applied. Hedging acts as the modifiers to a set, much like the way adjectives and adverbs modify statements in the English language.

Hedges are used to adjust the characteristics of the fuzzy set. In general, when a hedge is used to dilute a set, the set is expanded. When a set is intensified with a hedge, the set is compressed.

Table 1. Hedges and their effect on fuzzy set

Key Word	Effect on set characteristics
About, near close to approximately	Approximate the set
not	Complement the set
somewhat rather quite	Dilute the set
very extremely	Intensify the set

Applications

Bus Time Tables: There are schedules to predict the actual travel time on the bus. They use fuzzy logic because it is impossible to give an exact answer to when the bus will be at a certain stop. There can be accidents, abnormal traffic backups, or the bus could break down. An observant scheduler would take all these possibilities into account, and include them in a fuzzy formula for figuring out the approximate schedule.

Predicting genetic traits: Genetic traits are a fuzzy situation. Many traits can't be linked to a single gene. So only specific combinations of genes will create a given trait. Secondly, the dominant and recessive genes that are frequently illustrated with Punnet squares are sets in fuzzy logic. The degree of membership is measured by the occurrence of a genetic trait.

Temperature Control: It is to keep the room at the same temperature consistently. How much does a room have to cool off before the heat kicks in again? There in lies the fuzzy logic. It is determined by what the temperature is actually set to. Membership in that set weakens as the room temperature varies from the set temperature. Once membership weakens to a certain point, temp control kicks in to get the room back to the temp it should be.

Auto focus on a Camera: It takes fuzzy logic to figure out what is meant to be the main object of the photo based on multitudes of inputs. Perhaps the standard is to focus on the object closest to the center of the viewer. Maybe it focuses on the object closest to the camera. It is not a precise science, and cameras err periodically. This margin of error is acceptable for the average camera owner, whose main usage is for snapshots.

Medical Diagnoses: The diagnosis can only be some degree within the fuzzy set. Fuzzy Logic can explain diagnosis made out of a list of number of symptoms.

Antilock Braking system: The point of an ABS is to monitor the braking system on the vehicle and release the brakes just before the wheels lock. Two main factors determining this are the speed of the car when the brakes are applied, and how fast the brakes are depressed. There is, of course, a margin for error. It is the job of the ABS to be "smart" enough to never allow the error go past the point when the wheels will lock. In other words, it doesn't allow the membership in the set to become too weak.

Fuzzy logic can be used to control household appliances such as

1. Washing machines (which sense load size and detergent concentration and adjust their wash cycles accordingly)
2. Automobile and other vehicle subsystems,
3. Air conditioners
4. Cameras
5. Refrigerators.
6. Digital image processing, such as edge detection
7. Rice cookers
8. Dishwashers
9. Elevators
10. Video game artificial intelligence
11. Massage boards and chat rooms
12. Microcontrollers and microprocessors

Misconceptions and Controversies

Fuzzy logic is the same as "imprecise logic".

It is an organized and mathematical method of handling inherently imprecise concepts. The concept of "coldness" cannot be expressed in an equation, because although temperature is a quantity, "coldness" is not.

Fuzzy logic is a new way of expressing probability.

Fuzzy logic and probability refer to different kinds of uncertainty. Fuzzy logic is specifically designed to deal with imprecision of facts, while probability deals with chances of that

happening. Lotfi Zadeh has created a fuzzy alternative to probability, which he calls possibility theory.

Fuzzy logic is difficult to scale to larger problems.

The technical limitations of fuzzy logic can be expected in practice, and work on fuzzy controllers will also encounter several problems of scale. Today fuzzy logic is used to solve very complex problems in the AI area. Probably the scalability and complexity of the fuzzy system will depend more on its implementation than on the theory.

Need and utility of Fuzzy logic

The most cogent criticisms come from Haack. He argues that there are only two areas in which fuzzy logic could possibly be demonstrated to be "needed". The first is Truth and Falsity and second is Fuzzy system Utility. He then maintains that in each case it can be shown that fuzzy logic is not necessary.

It can not be used for new problems

Fuzzy logic cannot be used for unsolvable problems. This seems fairly reasonable, but its perception of being a guessing game may lead people to use it for anything.

It is not accurate

It is an obvious drawback to fuzzy logic. The results of fuzzy logic are perceived as a guess, so it may not be as widely trusted as an answer from classical logic.

It is confusing

Fuzzy logic can be easily confused with probability theory, and the terms used interchangeably. While they are similar concepts, they do not say the same things. Probability is the likelihood that something is true. Fuzzy logic is the degree to which something is true.

It is unnecessary

Classical logicians argue that fuzzy logic is unnecessary. Anything that fuzzy logic is used for can be easily explained using classic logic. Fuzzy logic claims that there can be a gray area between true and false. But classic logic says that the definition of terms is opposed to the actual truth of the statement. Haack believes fuzzy logic is not necessary because the calculations are more involved and partial membership values can be eliminated by defining terms more precisely.

It is unacceptable

The biggest problem with Fuzzy logic is low respectability. While fuzzy logic may be the superset of all logic, people don't believe it. Classical logic is much easier to agree with because it delivers precision. Open-mindedness is needed in order to change the acceptance of fuzzy logic.

Lack of Semantic Clarity

Haack's objections stem from a lack of semantic clarity, and the fuzzy statements may be translatable into phrases which classical logicians would find palatable.

CONCLUSION

Fuzzy systems, including fuzzy logic and fuzzy set theory, provide a rich and meaningful addition to standard logic. The mathematics generated by these theories is consistent, and fuzzy logic may be a generalization of classic logic. The applications which may be generated from or adapted to fuzzy logic are wide-ranging, and provide the opportunity for modeling of conditions which are inherently imprecisely defined, despite the concerns of classical logicians. Many systems may be modeled, simulated, and even replicated with the help of fuzzy systems.

Fox argues that Fuzzy and classic logics need not be seen as competitive, but complementary. Fuzzy logic can be used to solve the same types of problems that classical logic does. So it is less that fuzzy logic has actual limitations and more that it has perceived limitations. Lastly, Fox also states that despite the objections of classical logicians, fuzzy logic has found its way into the world of practical applications, and has proved very successful there. He maintains, pragmatically, that this is sufficient reason for continuing to develop the field. So, ultimately, Fox believes that the following three areas can benefit from fuzzy logic:

"Requisite" Apparatus- Use fuzzy logic to describe real-world relationships that are inherently fuzzy.

"Prescriptive" Apparatus- Use fuzzy logic because some data is inherently fuzzy and needs fuzzy calculus.

"Descriptive" Apparatus- Use fuzzy logic because some inferencing systems are inherently fuzzy.

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